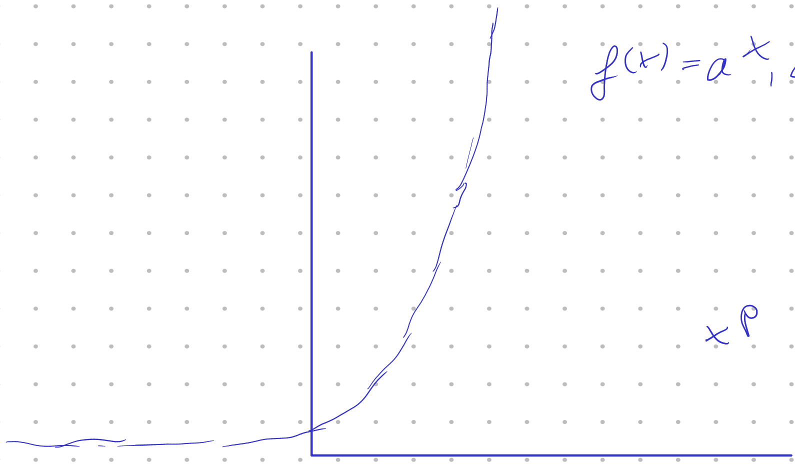
$f(x) = 10^x$



$f(x) = a^x,\ a > 1$

$f(0) = 10^0 = 1$

$f(1) = 10^1 = 10$

$f(2) = 10^2 = 100$

$f(3) = 10^3 = 1000$

genauso könnte man auch

$f(\frac{1}{2})$, $f(\sqrt{3})$, $f(3{,}7)$ usw. berechnen (aber nicht mehr im Kopf)

Umkehrfunktion: $f^{-1}(x) = \log_{10}(x)$

$f^{-1}(1) = 0$ (denn $10^0 = 1$)

$f^{-1}(10) = 1$ (denn $10^1 = 10$)

$f^{-1}(100) = 2$ (denn $10^2 = 100$)

$$\log(2) + \log(7x) + \log(2x+8) + \log(5-10x)$$

$$= \log(2 \cdot 7x \cdot (2x+8) \cdot (5-10x))$$

$$+\log(2) - \log(7x) - \log(2x)$$

$$= \log\left(\frac{2}{7x}\right) - \log(2x)$$

$$= \log\left(\frac{\frac{2}{7x}}{\frac{2x}{1}}\right) = \log\left(\frac{2}{7x} \cdot \frac{1}{2x}\right) = \log\left(\frac{2}{7x \cdot 2x}\right)$$

$$\log(2x+1) + \log(3) + \log(7x^2) - \log(8x-2) + \log(10)$$
$$- \log(\sqrt{x} - 1)$$
$$= \log\left(\frac{(2x+1)\cdot 3 \cdot 7x^2 \cdot 10}{(8x-2)(\sqrt{x}-1)}\right)$$

$$5\log(8x+1) = \log((8x+1)^5)$$

$$\log((x+2)^2) = 2\log(x+2)$$

$$-\log(x^2) = \log((x^2)^{-1}) = \log(x^{-2})$$
$$= \log\left(\frac{1}{x^2}\right)$$

$$2\log(x+1) - 3\log(2x^2) + 7\log(\sqrt{x}-1) + 5\log x$$
$$- 10\log(x^3+5)$$

$$= \log\left(\frac{(x+1)^2(\sqrt{x}-1)^7 x^5}{(2x^2)^3(x^3+5)^{10}}\right)$$

Es gibt keine Rechenregeln für

$\log(x+y)$ oder $\log(x-y)$

$\neq \log(x) + \log(y)$

$\log_2(32)$

$= \log_2(4 \cdot 8)$

$= \log_2(2^2 \cdot 2^3)$

$= \log_2(2^5)$

$= 5$

$\log_2(2^5) = 5$

$\log_4(32)$

$= \log_4(4 \cdot 8)$

$= \log_4(4 \cdot 4 \cdot 2)$

$= \log_4(4^2 \cdot 2) = \log_4(4^2) + \log_4(2)$

$= 2 + \log_4(4^{1/2})$

$= 2 + \frac{1}{2}\log_4(4^1)$

$= 2 + \frac{1}{2} \cdot 1$

$= \frac{5}{2}$

alternativ mit Basiswechsel

$\log_4(32) = \frac{\log_2(32)}{\log_2(\underbrace{4}_{=2^2})}$

$= \frac{5}{2}$

$\log_6(\sqrt[3]{6}) = \log_6(6^{1/3}) = \frac{1}{3}\log_6(6) = \frac{1}{3}$

$\log_3\left(\frac{1}{9}\right) = \log_3\left(\frac{1}{3^2}\right) = \log_3(3^{-2}) = -2\underbrace{\log_3(3)}_{=-2}$

$\log_a(\sqrt[q]{a^p}) = \log_a(a^{p/q}) = p/q \log_a(a) = p/q$

$$e^{x} - 4e^{-x} = 0$$

$$e^{x} = 4e^{-x} \quad | \log(-)$$

$$\log(e^{x}) = \log(4e^{-x})$$

$$x = \log(4) + \log(e^{-x})$$

$$x = \log(4) + (-x)\underbrace{\log_e(e)}_{=1}$$

$$x = \log(4) - x \quad | +x$$

$$2x = \log(4) \quad | :2$$

$$x = \frac{1}{2}\log(4)$$

$$x = \log(\underbrace{4^{1/2}}_{=\sqrt{4}})$$

$$x = \log(2)$$

$$32 \cdot 2^{x} = 64^{x} \cdot 16^{-x}$$

1. Mglk.: „Holzhamer"

$$32 \cdot 2^{x} = 64^{x} \cdot 16^{-x} \quad | \log(-)$$

$$\log(32 \cdot 2^{x}) = \log(64^{x} \cdot 16^{-x})$$

$$\log(32) + \log(2^{x}) = \log(64^{x}) + \log(16^{-x})$$

$$\log(32) + x\log(2) = x\log(64) + (-x)\log(16)$$

$$x \log(2) - x \log(64) + x \log(16) = -\log(32)$$

$$x \cdot (\log(2) - \log(64) + \log(16)) = -\log(32)$$

$$x \log\left(\frac{2 \cdot 16}{64}\right) = -\log(32)$$

$$x \log\left(\frac{1}{2}\right) = -\log(32)$$

$$x = -\frac{\log(32)}{\log(\frac{1}{2})}$$

$$x = -\frac{\log_2(32)}{\log_2(\frac{1}{2})}$$

$$x = -\frac{\log_2(2^5)}{\log_2(2^{-1})}$$

$$x = -\frac{5}{-1}$$

$$x = 5$$

2. Mglk.: Basis angleichen:

$$32 \cdot 2^x = 64^x \cdot 16^{-x}$$

$$2^5 2^x = (2^6)^x \cdot (2^4)^{-x}$$

$$2^{5+x} = 2^{6x} \, 2^{-4x}$$

$$2^{5+x} = 2^{2x} \qquad | \log_2(\ldots)$$

$$\log_2(2^{5+x}) = \log_2(2^{2x})$$

$$(5+x) \cdot \log_2(2) = 2x \log_2(2)$$

$$5 + x = 2x$$

$x = 5$

$$e^{x-1} = e^x - 1 \quad | \log(\sim)$$

$$\log(e^{x-1}) = \log(e^x - 1)$$

$$(x-1)\log(e) = \log(e^x - 1)$$

funktioniert nicht, da Differenz im Arg. des Logarithmus.

Besserer Weg:

$$e^{x-1} = e^x - 1$$

$$e^x \cdot e^{-1} = e^x - 1 \quad | - e^x$$

$$e^x \cdot e^{-1} - e^x = -1$$

$$e^x (e^{-1} - 1) = -1 \quad | : (e^{-1} - 1)$$

$$e^x = \frac{-1}{e^{-1} - 1} \quad | \log(\cdots)$$

$$\log(e^x) = \log\left(\frac{-1}{e^{-1} - 1}\right)$$

$$x = \log\left(\frac{-1}{e^{-1} - 1}\right)$$

$$= \log\left(\frac{e}{e-1}\right)$$

$$= \log(e) - \log(e-1)$$

$$= 1 - \log(e-1)$$

$$\frac{-1}{e^{-1}-1} = \frac{1}{1 - \frac{1}{e}} = \frac{1}{\frac{e-1}{e}} = \frac{e}{e-1}$$

alternativ: Substitution

$$e^{x-1} = e^x - 1$$

$$e^x e^{-1} = e^x - 1$$

subst.: $u = e^x$

$$u\, e^{-1} = u - 1$$

$$u\, e^{-1} - u = -1$$

$$u(e^{-1} - 1) = -1$$

$$u = \frac{-1}{e^{-1} - 1}$$

Umformung siehe oben

$$u = \frac{e}{e-1}$$

Rücksubst.:

$$e^x = \frac{e}{e-1} \quad | \log(\cdot)$$

$$x = \log\left(\frac{e}{e-1}\right)$$

$$x = 1 - \log(e-1)$$

$$\log(2x+1) = 5 \quad | e^{(\cdot)}$$

$$2x + 1 = e^5$$

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Umrechnung zw. Gradmaß und Bogenmaß

$$\frac{x}{360} = \frac{y}{2\pi}$$

x Winkel im Gradmaß
y Winkel im Bogenmaß

Bsp.: welchen Bogenmaß entspricht 30^0?

$x = 30$

$$\frac{30}{360} = \frac{y}{2\pi}$$

$$y = \frac{2\pi}{12} = \frac{\pi}{6}$$

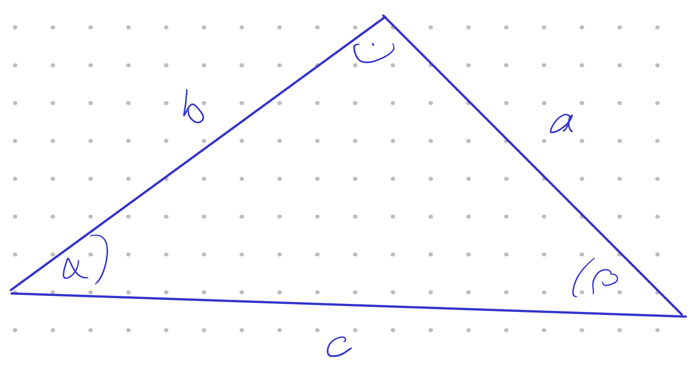
welchem Gradmaß entspricht $\frac{\pi}{4}$?

$$\frac{\frac{\pi}{4}}{2\pi} = \frac{x}{360} \quad | \cdot 360$$

$$\frac{\pi}{4} \cdot \frac{1}{2\pi} \cdot 360 = x$$

$$45 = x$$

Antwort: 45°



	a	b	c	α	β
a)	8	12	$\sqrt{208}$		
b)	7			45	
c)			5		60
d)		7	9		
e)			11	67	

Ber. von c mit Pythagoras:

a) $a^2 + b^2 = c^2 \Rightarrow c = \sqrt{a^2 + b^2}$

$$c = \sqrt{208}$$

Ber. von α mit Def. des Sinus:

$$\sin(\alpha) = \frac{a}{c} = \frac{8}{\sqrt{208}} \quad | \arcsin(\ldots)$$

$$\alpha = \arcsin\left(\frac{8}{\sqrt{208}}\right)$$