

Elementares Rechnen

1. a) $\frac{a^3 - 3a^2b + 3ab^2 - b^3}{a - b} = \frac{(a - b)^3}{a - b} = (a - b)^2 \quad \text{for } a \neq b$
 - b) $\frac{m^2 - 49}{6m - 42} : \frac{m^2 + 14m + 49}{12m + 84} = \frac{(m + 7)(m - 7)}{6 \cdot (m - 7)} \cdot \frac{12(m + 7)}{(m + 7)^2} = \frac{12}{6} = 2$
 - c) $\frac{m(m - 1)}{(m + 1)(m - 1)} + \frac{2m}{(m + 1)(m - 1)} + \frac{m(m + 1)(m - 1)}{(m + 1)(m - 1)} = \frac{m^2(m + 1)}{(m + 1)(m - 1)} = \frac{m^2}{m - 1}$
 - d) $1 - \frac{a - b}{a - b - a} = 1 - \frac{a - b}{-b} = \frac{b + a - b}{b} = \frac{a}{b}$
2. a) $\sqrt{2\sqrt{2}\sqrt{2}} = \sqrt{\sqrt{2^2 \cdot 2}\sqrt{2}} = \sqrt{\sqrt{\sqrt{2^6 \cdot 2}}} = 2^{\frac{7}{8}}$
 - b) $\sqrt{\sqrt{256}} = \sqrt{16} = 4$
 - c) $\frac{\sqrt{72}}{\sqrt{2}} = \sqrt{\frac{72}{2}} = \sqrt{36} = 6$
 - d) $\sqrt{\frac{18}{5} \sqrt{\frac{2}{5\sqrt{5}}} \sqrt{\frac{4}{5}}} = \sqrt{\frac{18}{5} \sqrt{\frac{2 \cdot 2}{5\sqrt{5} \cdot \sqrt{5}}}} = \sqrt{\frac{18}{5} \sqrt{\frac{4}{25}}} = \sqrt{\frac{18 \cdot 2}{5 \cdot 5}} = \sqrt{\frac{36}{25}} = \frac{6}{5}$
 - e) $\frac{\sqrt{108p^3}}{\sqrt{3p}} = \sqrt{36p^2} = 6p$
 - f) $\frac{\sqrt{16x^2 + 3\sqrt{(3x^2 + 3)(3x^2 + 3)}}}{\sqrt{25x^2 + 9}} = \frac{\sqrt{16x^2 + 3(3x^2 + 3)}}{\sqrt{25x^2 + 9}} = \frac{\sqrt{16x^2 + 9x^2 + 9}}{\sqrt{25x^2 + 9}} = \frac{\sqrt{25x^2 + 9}}{\sqrt{25x^2 + 9}} = 1$
 - g) $3^5 \cdot (3 - 8 + 5) = 0$
 - h) $4^{3 \cdot 2} = 4^6$
 - i) $a^{3n-n} \cdot b^{m-3m} = a^{2n} \cdot b^{-2m} = \left(\frac{a^n}{b^m}\right)^2$
 - j) $\frac{9^5}{12^4} = \frac{3^5 \cdot 3^5}{3^4 \cdot 4^4} = \frac{3^6}{4^4}$
 - k) $\frac{(p^2q)(pq^2r)}{(pqr)^2} = \frac{p^3q^3r}{p^2q^2r^2} = \frac{pq}{r}$
 - l) $\frac{6a^5c^{-7}d}{15a^{-2}b^{-5}c^3d^5} = \frac{6}{15}a^{5-(-2)}b^{-(-5)}c^{-7-3}d^{1-5} = \frac{2a^7b^5}{5c^{10}d^4}$
3. a) $(3m + 2n)^2 = 9m^2 + 12mn + 4n^2$
 - b) $x^2 - 6x + 9 = (x - 3)^2$

c) $4a^2 + 24ab + 36b^2 = (2a + 6b)^2$

d) $(2 + 3a^2)(2 - 3a^2) = 4 - 9a^4$

e) $d^2 - f^2 = (d - f)(d + f)$

f) $27x^3 - 27x^2y + 9xy^2 - y^3 = (3x - y)^3$