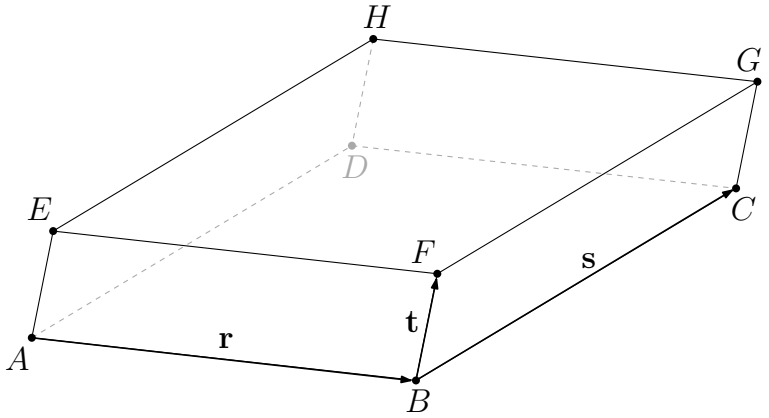
Vectors and systems of linear equations

1. Determine the distance between the following points **a** and **b** :

   a) $\mathbf{a} := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$; $\mathbf{b} := \begin{pmatrix} 5 \\ 1 \\ 6 \end{pmatrix}$ b) $\mathbf{a} := \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$; $\mathbf{b} := \begin{pmatrix} 6 \\ -2 \\ -2 \end{pmatrix}$

2. Let the following cuboid with vertices $A, B, \ldots, H$ be given. Let the coordinates of 4 vertices be known: $A = (-2, 1, 0)$, $B = (0, 5, -2)$, $C = (-7, 9, -1)$ and $F = (1, 6, 1)$



   a) Determine the vectors $\mathbf{r} = \overrightarrow{AB}$, $\mathbf{s} = \overrightarrow{BC}$ and $\mathbf{t} = \overrightarrow{BF}$.

   b) What is the volume of the cuboid?

   c) Determine the coordinates of the remaining vertices.

3. Determine a parameter representation of that straight line g which passes through the given points **a** and **b**.

   a) $\mathbf{a} := \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}$, $\mathbf{b} := \begin{pmatrix} 7 \\ 3 \\ 8 \end{pmatrix}$ b) $\mathbf{a} := \begin{pmatrix} 6 \\ 11 \\ 2 \end{pmatrix}$, $\mathbf{b} := \begin{pmatrix} 0 \\ 9 \\ -1 \end{pmatrix}$

4. The following systems of equations each have a unique solution (x, y, z). Determine this solution.

   a) $\left\{ \begin{array}{rrrl} x & +y & +z & = 6 \\ 2x & -y & +2z & = 6 \\ 3x & -2y & +z & = 2 \end{array} \right.$ b) $\left\{ \begin{array}{rrrl} 2x & +3y & -z & = 1 \\ x & +3y & +z & = 2 \\ -2x & -2y & +4z & = 4 \end{array} \right.$

5. Sketch the set of all solutions to the following systems of equations

a) $\left\{\begin{array}{rrl} x & +2y & =0 \\ 2x & +4y & =0 \end{array}\right.$ b) $\left\{\begin{array}{rrl} x & +2y & =2 \\ 2x & +4y & =4 \end{array}\right.$

6. Determine the positional relationship for the lines g_1 and g_2, where:

a) $g_1 = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$, $g_2 = \begin{pmatrix} -2 \\ 7 \\ 9 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

b) $g_1 = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix}$, $g_2 = \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 5 \\ -10 \\ 20 \end{pmatrix}$