Elementary algebra

1. a) $\dfrac{a^3 - 3a^2b + 3ab^2 - b^3}{a - b} = \dfrac{(a-b)^3}{a-b} = (a-b)^2$ for $a \neq b$

   b) $\dfrac{m^2 - 49}{6m - 42} : \dfrac{m^2 + 14m + 49}{12m + 84} = \dfrac{(m+7)(m-7)}{6 \cdot (m-7)} \cdot \dfrac{12(m+7)}{(m+7)^2} = \dfrac{12}{6} = 2$

   c) $\dfrac{m(m-1)}{(m+1)(m-1)} + \dfrac{2m}{(m+1)(m-1)} + \dfrac{m(m+1)(m-1)}{(m+1)(m-1)} = \dfrac{m^2(m+1)}{(m+1)(m-1)} =$
   $\dfrac{m^2}{m-1}$

   d) $1 - \dfrac{a-b}{a-b-a} = 1 - \dfrac{a-b}{-b} = \dfrac{b+a-b}{b} = \dfrac{a}{b}$

2. a) $\sqrt{2\sqrt{2\sqrt{2}}} = \sqrt{\sqrt{2^2 \cdot 2\sqrt{2}}} = \sqrt{\sqrt{\sqrt{2^6 \cdot 2}}} = 2^{\frac{7}{8}}$

   b) $\sqrt{\sqrt{256}} = \sqrt{16} = 4$

   c) $\dfrac{\sqrt{72}}{\sqrt{2}} = \sqrt{\dfrac{72}{2}} = \sqrt{36} = 6$

   d) $\sqrt{\dfrac{18}{5}\sqrt{\dfrac{2}{5\sqrt{5}}\sqrt{\dfrac{4}{5}}}} = \sqrt{\dfrac{18}{5}\sqrt{\dfrac{2}{5\sqrt{5}}\dfrac{2}{\sqrt{5}}}} = \sqrt{\dfrac{18}{5}\sqrt{\dfrac{4}{25}}} = \sqrt{\dfrac{18}{5}\dfrac{2}{5}} = \sqrt{\dfrac{36}{25}} = \dfrac{6}{5}$

   e) $\dfrac{\sqrt{108p^3}}{\sqrt{3p}} = \sqrt{36p^2} = 6p$

   f) $\sqrt{16x^2 + 3\sqrt{(3x^2+3)(3x^2+3)}} = \sqrt{16x^2 + 3(3x^2+3)} = \sqrt{16x^2 + 9x^2 + 9} =$
   $\sqrt{25x^2 + 9}$

   g) $3^5 \cdot (3 - 8 + 5) = 0$

   h) $4^{3 \cdot 2} = 4^6$

   i) $a^{3n-n} \cdot b^{m-3m} = a^{2n} \cdot b^{-2m} = \left(\dfrac{a^n}{b^m}\right)^2$

   j) $\dfrac{9^5}{12^4} = \dfrac{3^5 \cdot 3^5}{3^4 \cdot 4^4} = \dfrac{3^6}{4^4}$

   k) $\dfrac{(p^2q)(pq^2r)}{(pqr)^2} = \dfrac{p^3q^3r}{p^2q^2r^2} = \dfrac{pq}{r}$

   l) $\dfrac{6a^5c^{-7}d}{15a^{-2}b^{-5}c^3d^5} = \dfrac{6}{15}a^{5-(-2)}b^{-(-5)}c^{-7-3}d^{1-5} = \dfrac{2a^7b^5}{5c^{10}d^4}$

3. a) $(3m + 2n)^2 = 9m^2 + 12mn + 4n^2$

   b) $x^2 - 6x + 9 = (x - 3)^2$

c) $4a^2 + 24ab + 36b^2 = (2a + 6b)^2$

d) $(2 + 3a^2)(2 - 3a^2) = 4 - 9a^4$

e) $d^2 - f^2 = (d - f)(d + f)$

f) $27x^3 - 27x^2y + 9xy^2 - y^3 = (3x - y)^3$