Prep Course Mathematics

Inequalities

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Inequalities: equivalence transformations

Inequalities are written using the comparison signs $<$, $\leqslant$, $>$, $\geqslant$.

Important equivalence transformations:

- addition/subtraction: $a < b \iff a + c < b + c$
- multiplication with/division by a positive constant:
$$a < b \iff ac < bc \iff \frac{a}{c} < \frac{b}{c}$$
where $c > 0$
- multiplication with/division by a negative constant and flipping the comparison sign:
$$a < b \iff ac > bc \iff \frac{a}{c} > \frac{b}{c}$$
where $c < 0$.
- interchanging sides and flipping the comparison sign:
$$a < b \iff b > a$$
- taking positive powers: $a < b \iff a^p < b^p$
where $a, b, p > 0$.

The above transformations are also valid using the other comparison signs.

Solving inequalities

Inequalities typically have *infinitely many solutions*.

Those are typically calculated using one of the following **two approaches:**

- Use equivalence transformations to isolate the variable.
  Example:

$$2x + 3 > 7 \quad \Longleftrightarrow \quad 2x > 4 \quad \Longleftrightarrow \quad x > 2$$

- Solve the associated equation and then check values in between the solutions.
  Example: $x^2 + 2x - 1 < 2$