Solving equations

1. a) $x = 3$
   b) Simplified we have the equation $7x + 8 = 0$ with solution $x = -8/7$
   c) $x_1 = 0; x_2 = -\frac{3}{4}$
   d) $x = 2$
   e) $x_1 = 0; x_2 = -\frac{2}{3}; x_3 = 1$
   f) no solution

2. a) $x = 1$
   b) Squaring and solving the quadratic equation yields candidates $x = 0$ and $x = 3$. Probe shows that only $x = 3$ is a solution.
   c) Squaring and solving the quadratic equation yields candidates $x = 0$ and $x = 5$. Probe shows that only $x = 5$ is a solution.
   d) Squaring and solving the quadratic equation yields $x = 8$, which is confirmed by the sample as the solution.
   e) Squaring, simplifying and squaring again gives $x = 4$. The sample shows that this is indeed a solution.
   f) After squaring and simplifying once, we get $x\sqrt{1 + 8x} = x^2 + 2x$. One sees that $x = 0$ is a solution (by test this is confirmed). Now divide by x on both sides, then square again and solve the quadratic equation, which yields the two candidates $x = 1$ and $x = 3$. Again, the sample shows that both candidates are indeed solutions. So in total there are three solutions 0, 1 and 3.

3. Resolve the absolute values in each case by case distinctions, then solve the resulting equations and check that the solutions satisfy the inequality assumed in the observed case. This leads to the following solutions:
   a) $x = 1$ and $x = 7$
   b) $x = 2$, $x = -2$
   c) Since $|x| = 0$ is equivalent to $x = 0$, we can omit the absolute value marks and solve the resulting quadratic equation. This leads to the solutions $x = 1$ and $x = 2$.
   d) First, we observe that the equation is defined only for x different from -2 and -5 (otherwise there would be zero in the denominator). Now we make a case distinction to resolve the absolute value terms: Case $x + 4 \geq 0$ leads after multiplication of both denominators (each on both sides) to a linear equation with solution candidate $x = -\frac{11}{4}$. Since $-\frac{11}{4} + 4 \geq 0$ this is a solution.

Case $x + 4 < 0$, using the same procedure as in the previous case, leads to a quadratic equation that has no solutions. Thus $-\frac{11}{4}$ is the only solution.