Prep Course Mathematics

Integration

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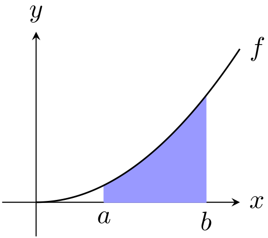
Motivation

Given $f\colon \mathbb{R} \to \mathbb{R}$.

Integral calculus: Area under f in $[a, b]$



Integral

$$\int_a^b f(x)\,\mathrm{d}x$$

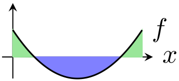
- a, b limits (or bounds) of integration
- f integrand
- x variable of integration
- $\int_a^b f(x)\,\mathrm{d}x = \int_a^b f(t)\,\mathrm{d}t = \int_a^b f(€)\,\mathrm{d}€$

Linearity

- For $\alpha, \beta \in \mathbb{R}$: $\int_a^b (\alpha f + \beta g)(x)\,\mathrm{d}x = \alpha \int_a^b f(x)\,\mathrm{d}x + \beta \int_a^b g(x)\,\mathrm{d}x$

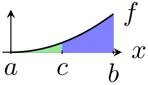
Positivity and monotonicity:

- $f \geqslant 0 \implies \int_a^b f(x)\,\mathrm{d}x \geqslant 0$
- $f \leqslant g \implies \int_a^b f(x)\,\mathrm{d}x \leqslant \int_a^b g(x)\,\mathrm{d}x$
- $\int_a^b f(x)\,\mathrm{d}x =$ area where $f \geqslant 0$ $-$ area where $f < 0$



Partition and absolute value:

- For $a < c < b$: $\int_a^b f(x)\,\mathrm{d}x = \int_a^c f(x)\,\mathrm{d}x + \int_c^b f(x)\,\mathrm{d}x$
- $\int_b^a f(x)\,\mathrm{d}x := -\int_a^b f(x)\,\mathrm{d}x$, and $\int_a^a f(x)\,\mathrm{d}x = 0$
- $\left|\int_a^b f(x)\,\mathrm{d}x\right| \leqslant \int_a^b |f(x)|\,\mathrm{d}x$



Determination of antiderivatives

"Differentiation is mechanics, integration is art."

$f(x)$	$f'(x)$
x^α	$\alpha x^{\alpha-1}$
e^x	e^x
$\ln\lvert x\rvert$	$\frac{1}{x}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\int f(x)\,\mathrm{d}x + c$	$f(x)$

linearity:

$$\int \alpha f(x) + \beta g(x)\,\mathrm{d}x = \alpha \int f(x)\,\mathrm{d}x + \beta \int g(x)\,\mathrm{d}x$$

products: ➪ integration by parts

$$\int f(x) g'(x)\,\mathrm{d}x = f(x)g(x) - \int f'(x) g(x)\,\mathrm{d}x$$

(special) quotients:
➪ partial fraction decomposition

(special) compositions: ➪ substitution

$$\int f(g(x)) g'(x)\,\mathrm{d}x = F(g(x)) + c$$