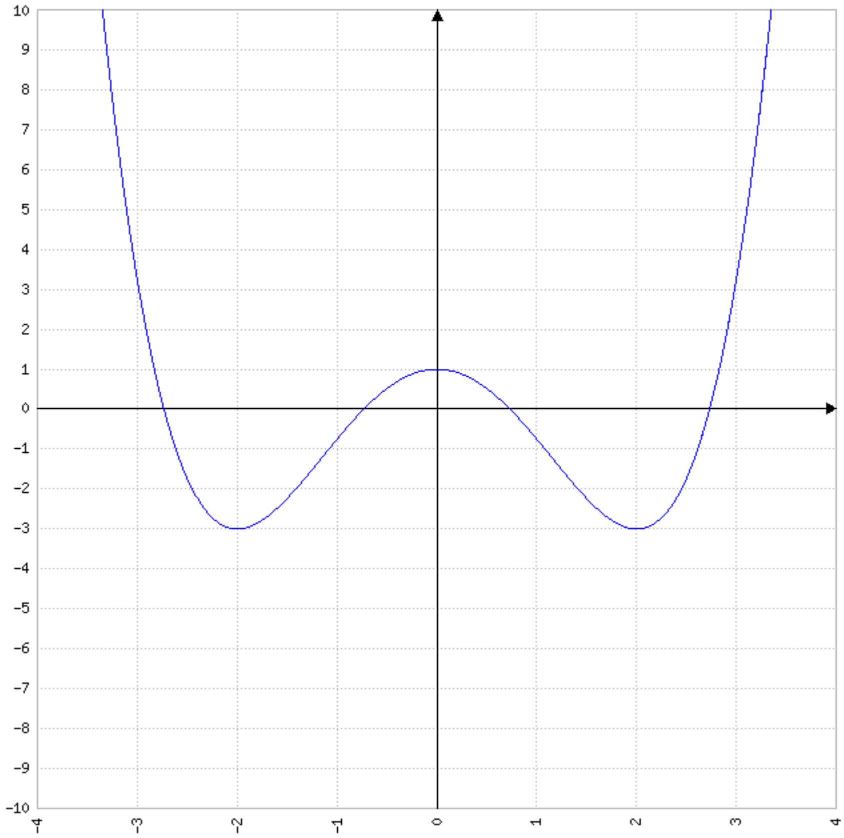
Differentiation

1. a) $f'(x) = 2$
   b) $f'(x) = -4x + 3$
   c) $f'(x) = -24x^3$
   d) $f'(x) = \dfrac{6}{x^3}$
   e) $f'(x) = (6x-1)(x+1) + (3x^2 - x)\cdot 1 = 9x^2 + 4x - 1$
   f) $f'(x) = \dfrac{-150x^4}{(6x^5+3)^2}$
   g) $f'(x) = 0,5(x-3)$
   h) $f'(x) = \dfrac{21}{10}x^{2.5}$
   i) $f'(x) = (4x^3 - 3x^2)\sqrt[3]{x} + \dfrac{x^4 - x^3}{3\sqrt[3]{x^2}}$
   j) $f'(x) = \left(\dfrac{5}{3} - \dfrac{2}{3x}\right)' = \dfrac{2}{3x^2}$
   k) $f'(x) = 15(5x+3)^2$
   l) $f'(x) = \dfrac{(-12x^2+2x)(x^2+3x-1) - (-4x^3+x^2-1)(2x+3)}{(x^2+3x-1)^2}$
   $= \frac{-4x^4-24x^3+15x^2+3}{(x^2+3x-1)^2}$
   m) $f'(x) = \dfrac{3}{\sqrt{6x-0.5}}$
   n) $f'(x) = \left(\dfrac{x^2-2x-3}{x^2-3x+2}\right)' = \dfrac{-x^2+10x-13}{(x^2-3x+2)^2}$
   o) $f'(x) = \dfrac{4}{3\sqrt[3]{\dfrac{1}{3}+2x}}$
   p) $f'(x) = 3(4x+1)\sin(2x^2+x)$
   q) $f'(x) = 4e^{4x-5}$
   r) $f'(x) = \dfrac{6x-2}{3x^2-2x}$
2. a) $f''(x) = -\dfrac{18}{x^4}$
   b) $f''(x) = 16\cdot e^{4x-5}$
   c) $f''(x) = -\cos^2(x)\cdot\cos(1-\sin(x)) - \sin(x)\cdot\sin(1-\sin(x))$
3. $f(x) = \dfrac{1}{4}x^4 - 2x^2 + 1$

- Derivatives:
  $f'(x) = x^3 - 4x$
  $f''(x) = 3x^2 - 4$
  $f'''(x) = 6x$
- maximum domain as well as range
  $\mathbb{D} = \mathbb{R}$
  $\mathbb{Y} = [-3; \infty)$
- Intersection with the y-axis: $f(0) = \frac{1}{4}(0)^4 - 2(0)^2 + 1 = 1$
- Zeros: $f(x) = \frac{1}{4}(x)^4 - 2(x)^2 + 1 = 0$
  $\Rightarrow x_1 = \sqrt{4+\sqrt{12}} \approx 2,73; x_2 = -\sqrt{4+\sqrt{12}} \approx -2,73;$
  $x_3 = \sqrt{4-\sqrt{12}} \approx 0,73; x_4 = -\sqrt{4-\sqrt{12}} \approx -0,73$
- Extrema: $f'(x) = x^3 - 4x = x(x^2-4) = 0$
  $\Rightarrow x_{e1} = 0; f''(0) = 3 \cdot 0 - 4 = -4 < 0 \Rightarrow$ local maximum at $x = 0$
  $x_{e2} = 2; f''(2) = 3 \cdot 2^2 - 4 = 8 > 0 \Rightarrow$ local maximum at $x = 2$
  $x_{e3} = -2; f''(-2) = 3 \cdot (-2)^2 - 4 = 8 > 0 \Rightarrow$ local minimum at $x = -2$
- Inflection points: $f''(x) = 3x^2 - 4 = 0$
  $\Rightarrow x_{i1} = \sqrt{\frac{4}{3}} \approx 1,15 \wedge f'''(\sqrt{\frac{4}{3}}) \neq 0 \Rightarrow$ Inflection point at $x = \sqrt{\frac{4}{3}}$
  $x_{i2} = -\sqrt{\frac{4}{3}} \approx -1,15 \wedge f'''(-\sqrt{\frac{4}{3}}) \neq 0 \Rightarrow$ Inflection point at $x = -\sqrt{\frac{4}{3}}$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