Vectors and systems of linear equations

1. a) $\left\| \begin{pmatrix} 5 \\ 1 \\ 6 \end{pmatrix} \right\| = \sqrt{5^2 + 1^2 + 6^2} = \sqrt{25 + 1 + 36} = \sqrt{62}$

   b) $\left\| \begin{pmatrix} 6 \\ -2 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 5 \\ 0 \\ -6 \end{pmatrix} \right\| = \ldots = \sqrt{61}$

2. a) $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$, $\mathbf{s} = \begin{pmatrix} -7 \\ 4 \\ 1 \end{pmatrix}$, $\mathbf{t} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

   b) Since a cuboid is present, we have

   $$V = \|\mathbf{r}\| \cdot \|\mathbf{s}\| \cdot \|\mathbf{t}\| = \sqrt{24} \cdot \sqrt{11} \cdot \sqrt{66} = \sqrt{17424} \overset{TR}{=} 132$$

   c) $D = (-9, 5, 1)$, $E = (-1, 2, 3)$, $G = (-6, 10, 2)$ and $H = (-8, 6, 4)$

3. a) $\left\| \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} \times \begin{pmatrix} 6 \\ 11 \\ 0 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 11 \\ -6 \\ -8 \end{pmatrix} \right\| = \sqrt{221}$

   b) $\left\| \begin{pmatrix} -4 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 7 \\ -3 \\ 2 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 5 \\ 15 \\ 5 \end{pmatrix} \right\| = 5\sqrt{11}$

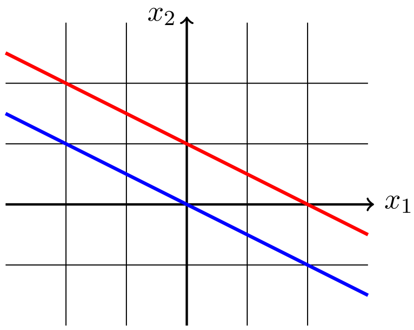
4. a) $g = \left\{ \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix} + \lambda \cdot \begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix} \ : \ \lambda \in \mathbb{R} \right\}$

   b) $g = \left\{ \begin{pmatrix} 6 \\ 11 \\ 2 \end{pmatrix} + \lambda \cdot \begin{pmatrix} -6 \\ -2 \\ -3 \end{pmatrix} \ : \ \lambda \in \mathbb{R} \right\}$

5. a) $x = 1$
   $y = 2$
   $z = 3$

   b) $x = 3$
   $y = -1$
   $z = 2$

6.



7. In order to determine the positional relationship, both representations can be equalized, resulting in a system of linear equations (with variables λ_1, λ_2):

   a)

$$\begin{aligned} \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} &= \begin{pmatrix} -2 \\ 7 \\ 9 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \\ \Leftrightarrow \lambda_1 \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} - \lambda_2 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} &= \begin{pmatrix} -3 \\ 4 \\ 10 \end{pmatrix} \end{aligned}$$

   Since the linear system has no solution and the direction vectors **are not multiples of each other**, the lines are skew.

   b)

$$\begin{aligned} \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} &= \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 5 \\ -10 \\ 20 \end{pmatrix} \\ \Leftrightarrow \lambda_1 \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} - \lambda_2 \begin{pmatrix} 5 \\ -10 \\ 20 \end{pmatrix} &= \begin{pmatrix} -2 \\ -6 \\ 1 \end{pmatrix} \end{aligned}$$

   Since the linear system has no solution and the direction vectors of the lines **are multiples of each other**, the lines are parallel.