Methods of proof

1. Prove the following statements:

    a) If $n \in \mathbb{Z}$ is even and $m \in \mathbb{Z}$ is arbitrary, then $n \cdot m$ is an even number.

    b) Let $n \in \mathbb{Z}$. Then $n + 1$ is even if and only if n is odd.

    c) If $n \in \mathbb{Z}$ is even, then n^2 is an even number.

2. Prove the following statements by complete induction.

    a) For all natural numbers n holds: $7^n - 4^n$ is divisible by 3.

    b) For all natural numbers $n \geq 5$ holds: $2^n > n^2$.