Prep Course Mathematics

Exponential and logarithmic functions

Dr. Simon Campese,   Dr. Dennis Clemens,   M. Sc. Yannick Mogge

TUHH   INSTITUTE OF **MATHEMATICS**

Exponential functions

- **Power function**: variable base, fixed power
  Examples: x^2, $\sqrt{x}$ etc.

- **Exponential function**: fixed base, variable power
  Examples: 2^x, $\left(\frac{1}{4}\right)^x$
  In general: a^x where $a > 0$

- The most important base is the Eulerian number $e = 2.718\ldots$
  (characterized by the fact that its associated exponential function e^x
  is its own derivative).

- Rules for powers apply without change.
  Example: $2^{x+3} = 2^x 2^3 = 2^x \cdot 8$
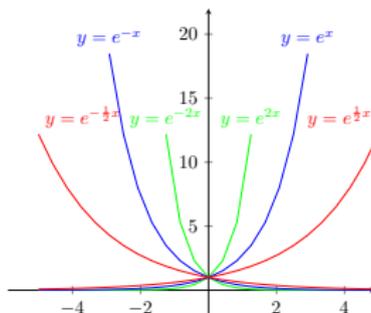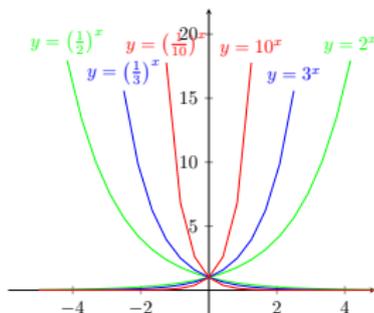
Logarithm
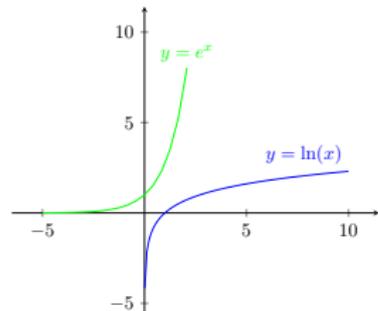
The logarithm $\log_a(x)$ „cancels"
the exponentiation a^x, i.e.

$$\log_a(a^x) = x \quad \text{and} \quad a^{\log_a(x)} = x.$$

For base $a = e$, the function $\log_e(x)$
has the shorthand $\ln(x)$.

Important rules

$$\log_a(x) + \log_a(y) = \log_a(xy)$$
$$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$$
$$\log_a(x^p) = p\log_a(x)$$

Change of base

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Equations involving exponential functions or logarithms

To solve equations in which the variable appears in an exponent or a logarithm, the same **approach** as for radical equations is used:

1. Isolate the logarithm or the exponential function in which the variable appears.
2. Cancel the function by taking exponentials or logarithms on both sides.
3. Repeat the first two steps until all exponentials or logarithms have been eliminated.
4. Solve the resulting equation.