Prep Course Mathematics

Trigonometry

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What is an angle?

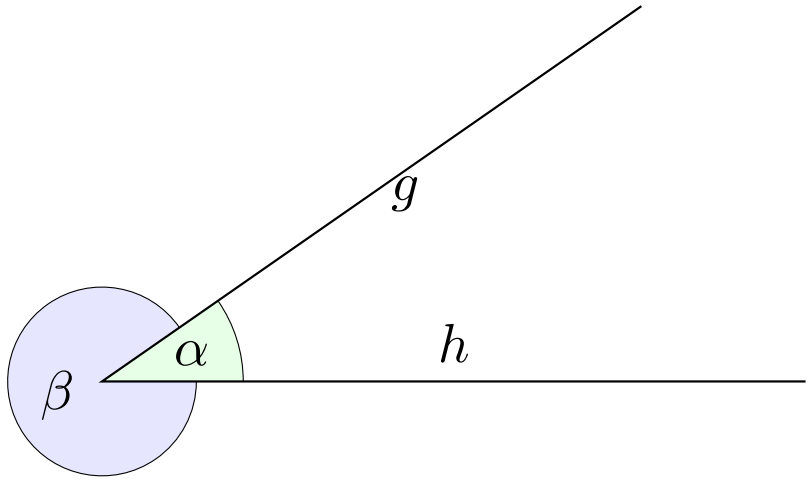


Figure: Angle between two half-lines (also called rays)

angle = measurement of rotation

angle measurements:

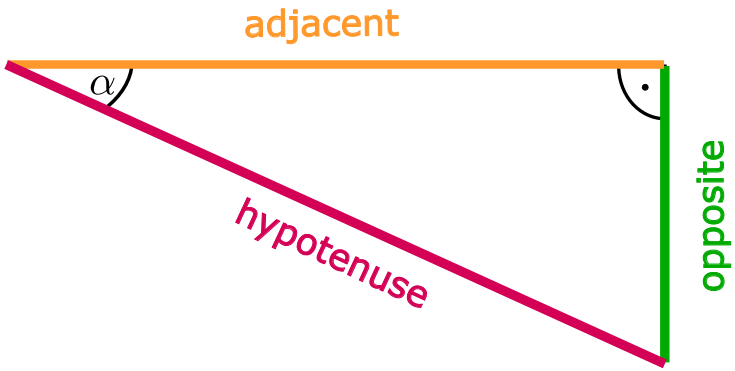
- degree
- radian

The sum of the interior angles in a triangle is always 180° (radian π).

Names of angles

	**full angle**	half lines overlap	360°
	**straight angle**	half lines form a straight line	180°
	**right angle**	half the size of straight angle	90°
	**acute angle**	smaller than right angle	
	**obtuse angle**	bigger than right angle, but smaller than straight angle	
	**reflex angle**	bigger than straight angle	

Right-angled triangle



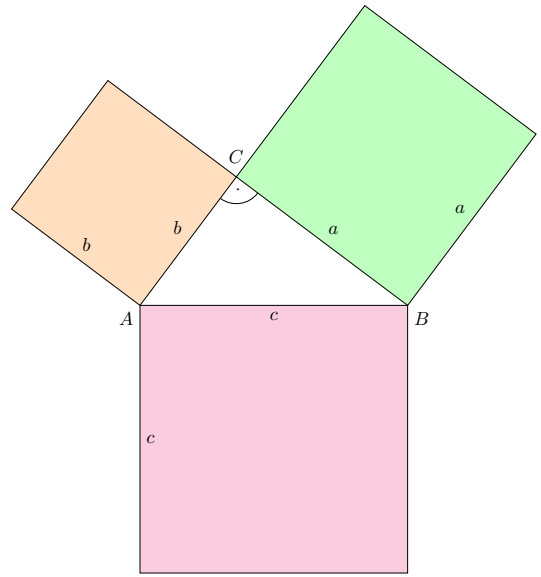
The ratio of these sides to each other defines the **trigonometric functions**.

$$\sin(\alpha) := \frac{\text{opposite}}{\text{hypotenuse}} \qquad \tan(\alpha) := \frac{\text{opposite}}{\text{adjacent}}$$

$$\cos(\alpha) := \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \cot(\alpha) := \frac{\text{adjacent}}{\text{opposite}}$$

Pythagorean theorem

$$a^2 + b^2 = c^2$$

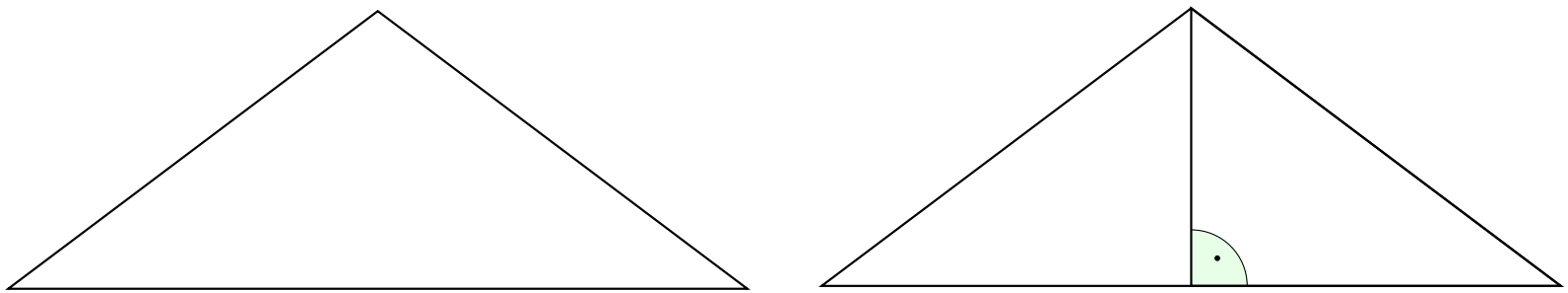


General triangle

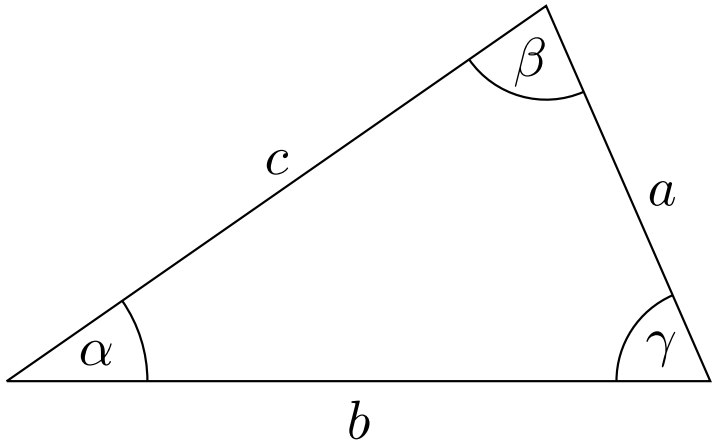
Based on our definition, we can calculate,
for example, the sine of an angle in a **general triangle**

$\Downarrow$

Drawing **right-angled auxiliary triangles**



General triangle



Further useful results for calculating side lengths or angles in general triangles:

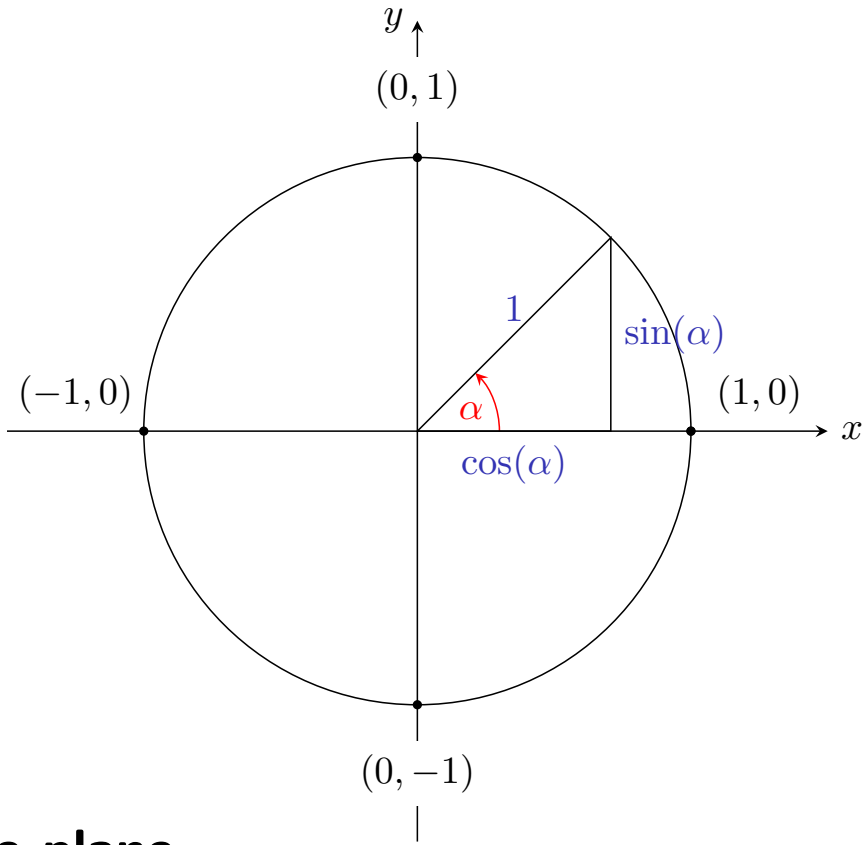
Sine rule:

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Cosine rule:

$$a^2 + b^2 - 2ab\cos(\gamma) = c^2$$

Unit circle: values for sine and cosine



Unit circle in the plane

$=$ circle with radius 1 around the points $(0,0)$

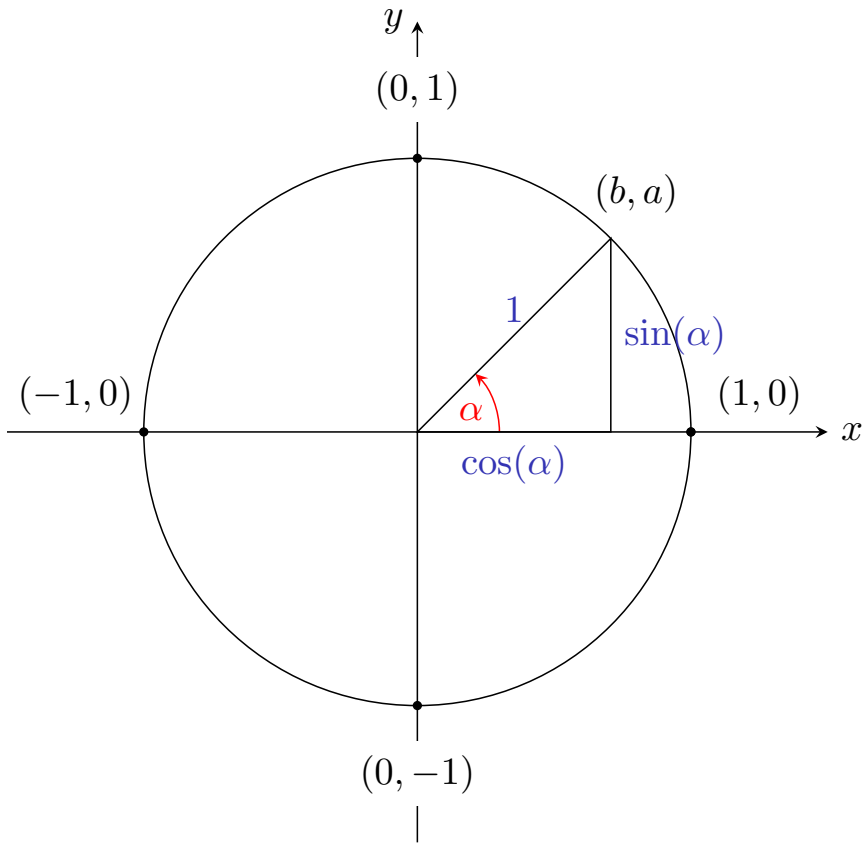
drawn triangle has a right angle

$\Downarrow$

Sine and cosine values can be read off:

- sine value of α on the y-axis
- cosine value of α on the x-axis

Angle measurement: radian



Starting point: unit circle in the plane

length of the complete arc
$=$ circumference of a circle with radius 1
$= 2\pi$

angle $\alpha =$ length of the arc between $(1,0)$ and (b,a)

Conversion from degree to radian

$$\frac{\text{degrees}}{360^\circ} = \frac{\text{radians}}{2\pi}$$

Let α be an angle which has as degree the form x°.
Then radian can be calculated as follows:

$$\alpha = \frac{x}{180} \cdot \pi.$$

degrees	0°	30°	45°	60°	90°	180°	270°	360°
radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

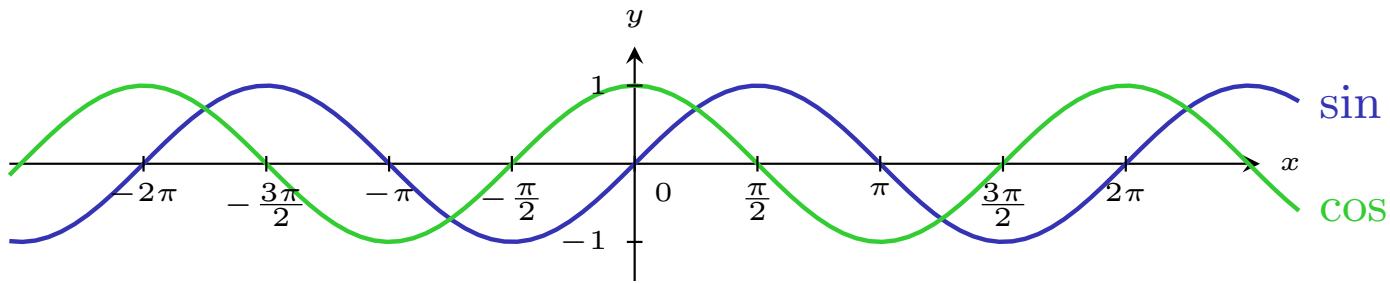
Trigonometric functions

Sine function

$$\sin\colon \mathbb{R} \to \mathbb{R}$$

Properties:

- antisymmetric,
  i.e. $\sin(-x) = -\sin(x)$
- zeros: $\{k \cdot \pi : k \in \mathbb{Z}\}$
- periodic with period 2π,
  i.e. $\sin(x) = \sin(x + 2\pi)$



Cosine function

$$\cos\colon \mathbb{R} \to \mathbb{R}$$

Properties:

- symmetric,
  i.e. $\cos(x) = \cos(-x)$
- zeros: $\{\frac{\pi}{2} + k \cdot \pi : k \in \mathbb{Z}\}$
- periodic with period 2π,
  i.e. $\cos(x) = \cos(x + 2\pi)$

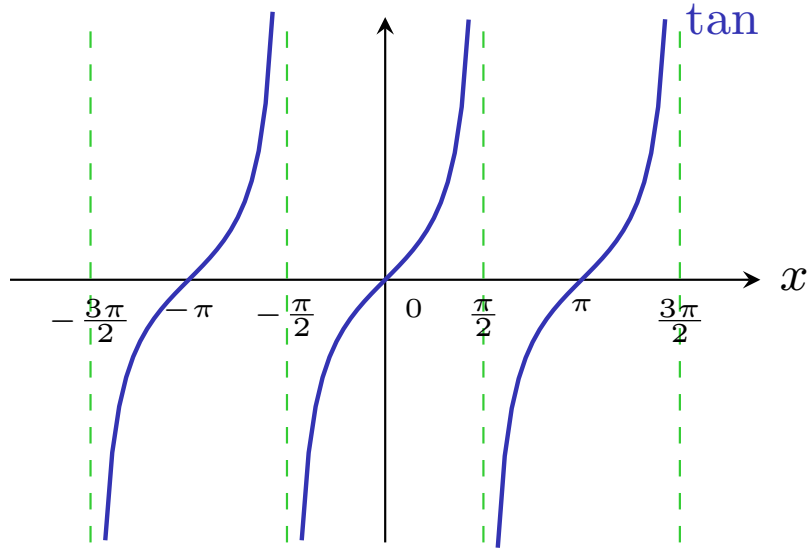
Trigonometric functions

Tangent function

$$\tan\colon \{x \in \mathbb{R} : \cos(x) \neq 0\} \to \mathbb{R}$$

Properties:

- antisymmetric
- periodic with period π, i.e. $\tan(x) = \tan(x + \pi)$
- zeros: $\{k \cdot \pi : k \in \mathbb{Z}\}$



Relations between trigonometric functions

- $\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$
- $\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$
- $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
- $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
- $\sin^2(x) + \cos^2(x) = 1$