Prep Course Mathematics

Elementary algebra

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TUHH

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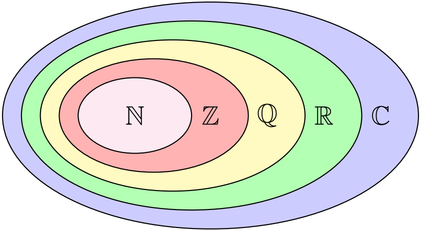
Numbers

You already know the following numbers:

- $\mathbb{N}$ (natural numbers)

- $\mathbb{Z}$ (integers)
- $\mathbb{Q}$ (rational numbers)
- $\mathbb{R}$ (real numbers)

Through elementary operations (addition, subtraction, multiplication and division) two numbers yield another number.



Conventions

Order of operations:

- Brackets first (from inside to out)
- Exponents
- Division and Multiplication (from left to right)
- Addition and Subtraction (from left to right)

⚠ There are some cases where brackets are implied by the notation:

- The fraction line denotes an operation that requires brackets, e.g.
  - $\dfrac{a \pm b}{c \pm d} = \dfrac{(a \pm b)}{(c \pm d)}$
- An exponent itself is always in brackets, e.g.
  - $a^{x \pm y} = a^{(x \pm y)}$
  - $a^{x \cdot y} = a^{(x \cdot y)}$
  - $a^{\frac{x}{y}} = a^{(\frac{x}{y})}$

Brackets

- To override operator precedence, brackets must be used:
- For simplifying a minus in front of brackets, the identity $-(\ldots) = -1 \cdot (\ldots)$ is useful.

Rules

- **Commutative law:**
  Summands or factors can be arbitrarily interchanged, i.e.

  $$a + b = b + a \quad \text{and} \quad a \cdot b = b \cdot a.$$

- **Associative law:**
  For summands or factors, order does not matter, i.e.

  $$(a + b) + c = a + (b + c) \quad \text{and} \quad (a \cdot b) \cdot c = a \cdot (b \cdot c).$$

- **Distributive law:**
  Multiplication and addition combine as

  $$a \cdot (b + c) = a \cdot b + a \cdot c.$$

Calculating with fractions

The following identities hold:

- Expansion/Simplifying: $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$

- Addition/Subtraction: $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$

- Multiplication/Division: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ and $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

Binomial expansion

The following identities hold:

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a+b)(a-b) = a^2 - b^2$

Pascals's Dreieck

$$
\begin{array}{lccccccccc}
n=0: & & & & & 1 & & & & \\
n=1: & & & & 1 & & 1 & & & \\
n=2: & & & 1 & & 2 & & 1 & & \\
n=3: & & 1 & & 3 & & 3 & & 1 & \\
n=4: & 1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

Powers

For a natural number n and a positive real number a,
✎ we write a^n as a shorthand for $\underbrace{a \cdot a \cdot \ldots a}_{n \text{ mal}}$.

More generally (but with a more complicated interpretation),
one can define a^p for an arbitrary real number p.

The following identities hold:

1. $(a^p)^q = a^{pq}$
2. $(ab)^p = a^p b^p$
3. $\dfrac{a^p}{b^p} = \left(\dfrac{a}{b}\right)^p$
4. $a^p a^q = a^{p+q}$
5. $\dfrac{a^p}{a^q} = a^{p-q}$

Roots

Roots are special powers: $\sqrt[p]{a} = a^{\frac{1}{p}}$.

⚠ $\sqrt{a}$ is only defined for $a \geqslant 0$
and is also greater than or equal to zero itself.

From the rules for powers we obtain:

1. $\sqrt[q]{\sqrt[p]{a}} = \sqrt[pq]{a}$
2. $\sqrt[p]{ab} = \sqrt[p]{a}\,\sqrt[p]{b}$
3. $\dfrac{\sqrt[p]{a}}{\sqrt[p]{b}} = \sqrt[p]{\dfrac{a}{b}}$

✎ We simply write $\sqrt{a}$ for the square root $\sqrt[2]{a}$.

Powers and roots combine well as the latter is just a special case of the former:

$$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p.$$

Calculating percentages

Fundamental identity: $$\frac{\text{percentage}}{100\%} = \frac{\text{percentage value}}{\text{base value}}$$